Registration No.: | | | | | | | | | | |

Total Number of Pages: 02

Course: IDD (B.Tech and M.Tech)
Sub_Code: 23BS1004

2^{nd} Semester Regular/Back Examination: 2024-25
SUBJECT: Mathematics - II
BRANCH(S): AE, AEIE, AERO, AUTO, BIOMED, BIOTECH, CE, CHEM, CIVIL, CSE, CSEAI, CSEAIML, CSEDS, CSEIOT, CSIT, CST, ECE, EEE, EEVDT, ELECTRICAL, ELECTRICAL & C.E, ELECTRONICS & C.E, CE, CSE, ECE, EE, ME
Time: 3 Hours
Max Marks: 100
Q.Code: S259
Answer Question No.1 (Part-I) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions: **(2 x 10)**

a) Give an example of a nonexact differential equation which have more than one integrating factors.

b) Write the general form of first order ordinary differential equations.

c) Find the particular integral of y'' + y = cos x.

d) Determine the Wronskian of $(t^2, 2t)^T$ and $(e^t, e^t)^T$

e) If $v = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$, then compute $\Delta \times v$.

f) Evaluate $\int_2^4 \int_0^1 x^2 y\,dx\,dy.$

g) Show that $F(z) = \bar{z}$ is not analytic

h) Find the residues of $f(z) = \frac{z^2+1}{z(z-1)}$

i) Give an example of a function which satisfy Laplace's equation.

j) Discuss the converse part of Cauchy's integral theorem.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) **(6 x 8)**

a) A body originally at 60^0 cools down to 40^0 in 15 minutes, when kept in air at a Temperature of 25^0. What will be the temperature of the body at the end of 30 minutes?

b) Solve the differential equation $(\sin x + \cos x \tan y)(dx + dy) + 2\sin y\,dy = 0$.

c) Show that $P_n(-1) = (-1)^n$

d) Solve the following initial value problem $x^2y'' - 3xy' + 4y = 0, y(1) = 1, y'(1) = 1$.

e) Apply Green's theorem to evaluate $\oint_C (2x^2 - y^2)dx + (x^2 + y^2)dy$, where C is the boundary of the surface in the xy-plane enclosed by the x-axis and the semi-circle $y = \sqrt{4 - x^2}$.

f) Find the directional derivative of $f(x, y) = \ln(x^2 + 3y)$ at the point $(1,1)$ in the direction of $\hat{i} + \hat{j}$

g) Find the divergence of $\vec{F}(x, y, z) = xyz\hat{i} + x^2y^2z^2\hat{j} + y^2z^3\hat{k}$.

h) Evaluate $\oint_C \frac{e^z}{(z-i)^4} dz$, where C is the circle $C: |z| = 2$ with positive orientation.

i) Define Cauchy-Riemann equations. Check whether the function $f(z) = \sqrt{|xy|}$ satisfies Cauchy-Riemann equations.

j) Evaluate $\int_{|z|=3} \frac{z+i}{z^2+4iz-4} dz.$

k) Evaluate the line integral $\int_0^{1+i} z^2 dz.$ Justify the answer.

l) Using Cauchy's integral formula evaluate $\int_{|z-1|=2} \frac{1}{z^2+1} dz.$

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four) **(16 x 2)**

Q3 a) Find the power series solution of the differential equation $y'' - 2y' + y = 0.$ **(8 x 2)**

b) Solve the differential equation $y' + x^5y = x^5y^7.$

Q4 a) Solve the differential equation $y'' - 2y' + y = xe^x \log x$, $x > 0$ by using method of Variation of parameter. **(8 x 2)**

b) Solve the differential equation $y'' + 2y' + 4y = \cos 4x$ by using the method of undetermined coefficients.

Q5 a) Evaluate $\iint_R (x^2 + y^2)dxdy$, where R is the region is bounded by $x = 0, y = 0, x + y = 1$. **(8 x 2)**

b) Show that the function $e^x(\cos y + i \sin y)$ is analytic and find its derivative.

Q6 a) Evaluate the integral $\int_{|z-3|=1} \frac{z^2}{(z-1)^3(z-2)} dz$ using residue theorem. **(8 x 2)**

b) Develop $f(z) = \frac{2z-3i}{z^2-3iz-2}$ in a Laurent series valid for $1 < |z| < 2.$